TEACHERS' INVOLVEMENT AND LEARNING IN A LESSON STUDY[1]

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The aim of this study is to analyse the way a group of basic education teachers got involved in a lesson study and their views, *as they discuss the features of tasks, students' expected difficulties and how to conduct exploratory work in the mathematics classroom. The conceptual framework draws on notions of task design and considers the features of lesson study. Data collection was made through a research journal made by one of the authors in the role of observer, video recording with transcription of the sessions, teachers' written reflections and interviews. The results underscore the value of focusing on students' reasoning in working in mathematical tasks as well as the need to address affective issues, regarding the way teachers are invited to become involved and participate in lesson studies.*

INTRODUCTION

Exploratory mathematics teaching is receiving an increasing support in international curriculum orientations for mathematics education (e.g., NCTM, 2000). In this approach students are called to deal with tasks for which they do not have an immediate solution method (closed problems and open problems). This represents an important departure from the didactical tradition in which the teacher just presents tasks that the students were already taught how to solve. In this tradition, the teacher begins by demonstrating the solution method and after presents the tasks for the students to practice it. With exploratory teaching, quite on the contrary, to solve the proposed tasks the students have to construct their own methods using their previous knowledge.

Exploratory work in the mathematics classroom creates opportunities for students to build or deepen their understanding of concepts, procedures, representations and mathematical ideas. Students are thus called to play an active role in the interpretation of the questions proposed, the representation of the information given and in designing and implementing solution strategies, which they must be able to present and justify to their colleagues and to the teacher. However, conducting exploratory mathematics teaching is a serious challenge for teachers, demanding specific knowledge, competency and disposition. In this paper we analyse the way a

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group of grade 5 and 6 teachers got involved in a lesson study and their emerging views on students' learning as they discuss the features of tasks and consider the possibilities for students' exploratory work in the mathematics classroom.

TASKS IN THE MATHEMATICS CLASSROOM

If mathematics teaching is mainly based on teacher lecturing, the concept of task is of little use. On the contrary, if mathematics teaching values the active role of students, this concept is essential, since tasks are a critical organizing element of the students' activity. However, tasks may play a variety of roles. The main goal of some tasks is to support learning, others are used to verify students' learning (assessment tasks), and, finally, others aim to get a deep understanding of students' capabilities, thinking processes and difficulties (research tasks).

Pólya (1945) draws a distinction between exercises and problems. Expanding this distinction, Stein and Smith (1998) present a typology of classroom tasks based on students learning. They distinguish between tasks with low and high level of cognitive demands. In the low-level cognitive tasks they make a distinction between: (i) "memorization" and (ii) "procedures without connections". In tasks with high cognitive demand, they distinguish between: (iii) "procedures with connections" and (iv) "doing mathematics". Stein and Smith illustrate these different categories using examples such as tasks that request students to define the relationship between different forms of representing a number as a fraction, decimal or percentage.

Skovsmose (2001) compares "exercises" with "landscapes for investigation", which include project work. Landscapes for investigation invite students to ask questions and seek explanations. However, it is necessary that students accept the teacher's challenge. The author also states that tasks rely on three major types of references – mathematics, real life, and "semi-reality" – i.e., situations that seem real, but it fact they are artificial and exclusively designed for learning. Based in these notions, the author identifies six types of learning environments but warns that the dividing lines between these environments are fluid and so students often move between them. He also points out that mathematics education should not be placed exclusively in an environment, but rather it should facilitate movement across them. In his view, the difficulties that such work creates to teachers, forcing them to modify the "didactic contract" implicitly established with students, places teachers on a "risk zone". As the author indicates, to work in such way teachers should look for support from their colleagues, working in collaborative settings.

Ponte (2005) indicates that tasks have two fundamental dimensions: mathematical challenge and structure. The degree of mathematical challenge (low/high) depends on the perception of the difficulty for a given person. On the other hand, the degree of structure (open/closed) refers to the nature of the statements regarding givens, goals, and conditions, which may be more or less detailed and open to interpretation. Crossing the two dimensions, we obtain four types of task: exercises are closed tasks presenting low mathematical challenge, problems are also closed tasks, but with a high degree of mathematical challenge, investigations are open tasks that present high mathematical challenge, and explorations are relatively open and have some challenging features for most students.

When teachers plan their work, they usually consider various types of tasks. Ponte (2005) suggests that diversification is necessary because each type of task plays a specific role in learning. Closed tasks (such as exercises and problems) are important for students' development of mathematical reasoning, which is based on a strict relationship between givens and results. Tasks with a small degree of challenge (as explorations and exercises) facilitate students achieving high rates of success and promote their self-confidence. More challenging tasks (as investigations and problems) are crucial to provide students deep mathematical experiences. Finally, open tasks are essential to help students to develop certain capacities, such as autonomy and ability to deal with complex situations.

This author points out that a certain task can be an exploration or an exercise, depending on students' prior knowledge. He also indicates that, in mathematical work, students mobilize knowledge they built outside of the school context. Contrary to the idea that students cannot carry out a task if they have not been taught directly how to solve it, the author notes that students learn out of school much knowledge they can mobilize in the mathematics classroom – and that is what exploratory tasks seek to promote. He also values students (re)discovery of a method for solving a question, and points out that this is often the best way to learn.

Ponte (2005) also considers that it is possible to diversify tasks by attending to context and the complexity of the work. The teacher should propose tasks in real contexts, so that students realize how mathematics is used in such contexts, and to take advantage of their knowledge of these contexts (application and modeling tasks). However, students may also feel challenged by tasks formulated within mathematical contexts (investigations, problems, explorations). By solving these tasks students may also understand how professional mathematicians develop their mathematical activity. In his view, in addition, tasks must provide a process of consistent learning, that facilitates students' construction of concepts, understanding of procedures, as well as increased knowledge of relevant representations and of connections within mathematics and within others domains. In order to achieve this goal, teachers must make decisions, define educational journeys and select tasks carefully. So, more than isolated tasks, teacher must organize sequence of tasks.

TEACHERS' PROFESSIONAL LEARNING IN A LESSON STUDY

Lesson study is a process to foster teachers' professional development. Recently it has gained international attention and it has been increasingly used in different grade levels, including higher education. A very important feature is that lesson studies take place within the school environment, where teachers play a central role. Usually, a lesson study begins with the identification of a relevant issue related to students' learning. Afterwards, the participants plan a lesson together considering the curriculum guidelines. They also predict students' difficulties, anticipate questions that might emerge in the classroom, formulate teaching strategies, and prepare the instruments to observe the lesson. The lesson is taught by one of the teachers with the others observing and taking notes paying special attention to students' learning. When the lesson is over the teachers meet together to analyse and reflect on the observed lesson. The analysis may lead to the reformulation of the lesson plan, to a change in the strategies and materials used, in the tasks proposed, in the questions asked to the students, etc... Frequently, a revised lesson is later taught by another teacher to other class, in cycles that may be repeated several times (Lewis, Perry, & Hurd, 2009; Murata, 2011). A central aspect of lesson studies is that they focus on students' learning and not on teachers' work. This distinguishes lesson studies from other observation processes which focus mainly on what teachers do. Indeed, lesson studies aim to examine students' learning and to observe, up close, the way they learn. When participating in lesson studies, teachers can learn about important professional issues, in relation to the teaching subject, curriculum guidelines, students' processes and difficulties, and classroom dynamics.

Undertaking a lesson study has many problematic features, concerning the way teachers regard their own work in the classroom and how they relate to each other and to outsiders. A lesson study may challenge the notion that teachers have how a class must be organized, what tasks may be proposed, how they may be conducted. Requiring teachers to work together, it may give raise to personal conflicts and difficulties in handling criticisms. That is, lesson studies may provoke many uncomfortable moments in participating teachers. Reporting on a series of lesson studies involving novice teachers, Carter, Gammelgaard and Pope (2006) indicated that "Throughout the process each year, participants report a variety of feelings. Journal entries over the years have included statements of fear, excitement, anger, elation, weariness, frustration, and success" (p. 131).

Lesson studies are meant to be collaborative endeavors. Participants may create a close relationship, getting ideas from each other but also mutual support (Boavida & Ponte, 2002). In this way, lesson studies create a context not only for teacher reflection but also for the development of the sense of confidence that is central to teacher development. As Hargreaves (1995) indicates:

> While reflection is central to teacher development, the mirror of reflection does not capture all there is to see in a teacher . . . However conscientiously it is done, the reflective glance can never quite get to the emotional heart of teaching . . . Understanding the emotional life of teachers, their feelings for and in their work, and attending to this emotional life in ways that positively cultivate it and avoid negatively damaging it should be absolutely central to teacher development efforts. (p. 21)

This close attention to didactical, mathematical, curricular, educational, and even political factors that are present in teachers' activity and professional development has to take into account all emotional aspects that may emerge and even get control over the educational processes.

RESEARCH METHODOLOGY

This research is conducted in the context of a lesson study carried out in a cluster of schools in Lisbon. This lesson study had its origin in a project developed by the cluster for which the principal asked the collaboration of the Instituto de Educação da Universidade de Lisboa (IE) to promote the professional development of the mathematics teachers. We proposed to carry out several lesson studies with teachers at different school levels and its was agreed that one of them would be undertaken with a group of grades 5-6 teachers (Inês, Francisca, Luísa, Maria, and Tânia) invited by the director of the cluster that also designated one of these teachers (Maria) as leader of the group. In a first preparatory meeting (with the participation of Maria) it was decided that the lesson study would concern a grade 5 topic (this grade was taught by three teachers of the group) since at this grade there was a new 2013 syllabus being applied and that could help the teachers to understand better how to deal with this syllabus, which was for them a major concern.

The first lesson study session was led by three members of the IE team (João Pedro, Marisa, and Joana) and the remaining sessions by two members (Marisa and Joana). These sessions usually take place every two weeks. In the sequence, we analyse four of the eight planned sessions that illustrate the working setting in this kind of teacher professional development context. Session 1 sought to present lesson study to the whole group of teachers, sessions 2 to 6 to deepen their knowledge about a topic and prepare a lesson on that topic, session 7 to observe a lesson, and session 8 to reflect on the observed lesson and on the whole lesson study process. Data collection is made through a research journal made by a member of the IE team in the role of observer, video recording with transcription of the sessions. We also collected but do not analyse here are teachers' written reflections and responses to interviews.

Data analysis has begun by identifying critical moments in sessions 1-4, looking at session transcripts and, when useful, to the video recording. We then categorized the identified episodes according to features that we considered of interest, regarding teachers' involvement in the session activity as well as regarding the key didactical issues emphasized in our conceptual framework: the nature of tasks, the identification of students' difficulties, attention to students' representations and students' reasoning. From this set of episodes we selected those that appeared to us more telling about the way this lesson study unfolded and about the teachers' participation, which we seek to interpret and analyse with our theoretical lenses.

RESULTS

A difficult beginning

The first session of the lesson study was mostly a presentation of the participants and of the work to be carried out. A most salient part of this session was the noticeable resistance from the teachers regarding the proposal to do a lesson study. Their participation in the lesson study was not their initiative – they were invited by the director of the cluster of schools. In the presentation meeting we had explained to

Maria, the coordinator of the group, what a lesson study was, and got the idea that she was convinced about its value and feasibility, but in this session she led the group in asking many questions about it. For example, she asked why to concentrate on just one topic if there is a new mathematics syllabus with many new features that the teachers are not sure how to deal with. We argued back that it was more productive to concentrate in just one topic in depth than to look superficially at many topics. She and other teachers as well, indicated to be uneasy with the idea of their classes being observed by others. We replied saying that in these observations the focus of interest was the students' reasoning and not the teacher's actions, but they did not seem very convinced. There were also questions regarding the attitude of the students in the presence of external observers. We indicated that in our former experiences in lesson study that was not a problem at all. Our arguments were not completely convincing but, finally, the proposal for the overall plan of the lesson study was approved by the teachers. As the topic to focus the attention in this study, the teachers chose rational numbers. In a later session they would define in a more precise way that the lesson to observe would concern teaching ordering, comparing, and equivalence of rational numbers. When the session was over, our team had the impression that this lesson study was going to be a disaster.

Identifying students' difficulties in mathematical tasks

In session 2, among other activities, we presented the teachers with a set of tasks, including exercises, problems, and explorations, and invited them to discuss the characteristics of these tasks and possible students' difficulties. This provided a good opportunity for teachers to reflect about different features of tasks.

We presented the taxonomy of tasks as exercises, problems, explorations and investigations. The first two terms were familiar to the teachers and they used them a lot during this whole session ("exercise", 19 times; "problem", 48 times), whereas they never used the terms "exploration" or "investigation". We may conclude that these two terms are not part of their usual professional vocabulary.

The last aspect of the presentation of the taxonomy on tasks, was a comment that a task is not a problem because it has a story attached to it, but because the students do not have a ready to use solution method. If they have such method, the task is just a simple exercise. This notion was a surprise for Maria, for whom a question with a story was a problem, regardless of its difficulty:

> Maria: When they have all the data that means it is an exercise…? [So an exercise] is an application, it is an application of knowledge, and not . . . Whereas a problem is a little bit more than that, they have to discover something… They have to apply what they already know.

This new notion of problem seemed to make sense to Maria and to the other teachers, and, slowly, began to inform the discussion in the lesson study.

The first reaction regarding the tasks proposed for analysis, from Inês, was that "this kind of exercises is quite advanced". This teacher argued a lot for her position, and got support from several others, like Luísa and Francisca. However, at some point, Luísa and Maria began saying that some of their students could solve one or another task:

> Maria (in question 2): If they transform everything in decimal numbers…
>
> Luísa: Yes.
>
> Marisa: Exactly.
>
> Francisca: It is very easy.
>
> Tânia: From here they go on very well.
>
> Maria: Yes.
>
> Luísa: Yes.
>
> . . .
>
> Luísa: Leaving aside question 3.2, that I think that they would get [the answers], the others no. And the 25% of the figure, some [students] I think would not get it.
>
> Maria: Most [students] would not do them [the tasks]. I have one or another [student] who would do.
>
> . . .
>
> Maria: Everything involving decimal numbers they would get it.
>
> Luísa: Yes, maybe.
>
> Maria: Perhaps they would get there. So, they would count . . .
>
> Tânia: To do this one they would need to have the notion of equivalent fractions… If they have the notion of equivalent fractions [they can solve it]…

This was the beginning of the discussion of the full set of tasks. In the sequence, as the teachers considered every task in turn, they found many cases in which their students could possibly solve the proposed questions. Inês still argue in several moments that this set of tasks would fit better grade 6 students, and it would be very difficult to grade 5 students. However, as the tasks were discussed one by one, the other teachers found many situations that their students could have ease to do and others that, whereas not so easy, they still could perhaps handle.

Later on in this session, Tânia made an important point about the role of representations:

> With the representation it [this task] is quite simple because when we do the representation and after we say that it is ⅓ or it is ⅖ of that, so we divide, all right? And then we say that it is only ¼ of that, it is easy! Now, without the representation it is not so

easy for them to do [the question]. What they usually do is: they multiply and divide. Because they have that . . . Usually grade 6 students do it this way.

When Tânia says "representation" she means "pictorial representation". This teacher voiced an important idea, that the difficulty of a task depends on the representation in which the task is formulated, and teachers should promote the use of pictorial representations along with the more formal representations of rational numbers as fractions and decimal numbers.

The tasks in which a part is given and questions are made about other parts promoted a lot of discussion among teachers. For students that do not know operations with rational numbers, such tasks require them to do two steps – first to construct the unit, given a part, and then find the required new part. The teachers had to think for a while how to handle such tasks themselves:

Maria: How would you approach this? I am sorry! This is ¾ and now how would you ask ½? How do they…?

Marisa (IE): What could be the first step?

(silence for 5 seconds)

Maria: Any suggestion?

Inês: It is to add a little bit that is missing.

Marisa (IE): First they need to understand what is then the…

Teachers: [All at the same time] The unit!

Dealing with tasks that had some elements of challenge created some excitement on teachers. For example, at some point in this session, during the discussion of another problem, Maria commented

But this is interesting, I like it a lot: the construction of the unit. To begin from here to construct the whole, and then the representations – having the unit to be divided by three or to be divided by two...

It became apparent early in this session that the teachers were not used to the meanings of terms that are quite important to speak about features of tasks, neither to distinguish among different kinds of tasks. For example, to mean "problem", Inês speaks of "advanced exercises". We decided to introduce some elements of this vocabulary and then use them informally expecting that the teachers to appropriate it.

An important feature of the discussion was the encouragement to teachers to make connections to their own professional experience. Some of the teachers – especially Maria and Tânia – made this several times, with great excitement. In other occasions, it was Marisa (from the IE team) that reported on her experience showing how her students managed tasks that the participating teachers found too difficult for their students. This close relation to professional practice brought an important element of authenticity that seemed to be powerful in letting the teachers to reflect on the issues under discussion.

During this discussion there were several opportunities for the teachers to see what the guidelines of new syllabus were. They were quite surprised with things that they did not notice, and all of them were very critical about some issues such as task with a subtraction of two mixed numbers in which the fraction of the number to be subtracted is small than the other fraction. The wide consensus that the teachers achieved regarding these curriculum documents appeared to be reassuring for them and provide some confidence to work with this new approach from a critical stance.

However, it also appeared that these teachers are not much used to propose tasks with challenging features, fearing that the students simply cannot solve them. A very important point is that, in this session, at least some of the teachers appeared to start thinking that challenging tasks could also be presented to students, at least to some of them, given the appropriate conditions. They could see that grade 5 students (at least some of them) could handle tasks that went beyond what they usually propose, with positive results. This attention to the features of tasks and to students' difficulties in solving them appeared to be a new notion to several teachers of the group that got very involved in many moments of the session, showing their excitement (as we illustrated with a comment from Maria). What looked like a disaster lesson study became at this point a very promising professional development activity.

Identifying students' achievements

In session 3 the teachers designed a set a diagnostic tasks to know about students' knowledge on rational numbers. One of the aims of session 4 was to analyse the results. Seeking to overcome the teachers' usual tendency to just focus on students' difficulties, Marisa began by asking the teachers about situations in which they were positively surprised with students' work. She had been to Maria's classes where the diagnostic test was administered and had analysed the results of one class. She offered herself to begin by pointing aspects that she found very interesting and gave several examples of interesting achievements. However, when it was the teachers' turn, what emerged was again their focus on students' difficulties. For example, Francisca said:

> Regarding [my class] children painted with ease the fractions, but many times they did not made the fraction representation. They only read a half, that's it. After, in this [question 3], they had more difficulty exactly in ¼ and in 1/8. It was very difficult for them.

Francisca goes on saying that students managed to paint with ease a half and one third of the pictures presented to add in the sequence a new set of difficulties. Marisa insisted in reorienting the discussion towards the positive surprises:

> Marisa: Perhaps we do the surprises first and then the difficulties.

Francisca: Surprise, surprise, was in exercise 4. They were easily able to get ¼ of the chocolate. I found that very cute because they already know how to do the computation [4/4 – 3/4 = 1/4], I was not expecting that.

In this way, Francisca referred some positive aspects of students' work. However, in her second intervention, she came back to difficulties:

Francisca: So, they made this, I found... This is where I found that that they had more difficulties.

Marisa: Difficulties, no. Surprises. Any other [question] that you were expecting that they would not solve?

Francisca then mentioned another surprise in her students' solutions, referring the knowledge that students have about the different representations of rational number:

Francisca: My surprises were really here in task 4. I found that fantastic. This representation of fraction, decimal and percent, that I thought that most of them would not be able to do, and most did.

Tânia is the last teacher to present her surprises in her students' work. She began by addressing these aspects perhaps having figured out the implicit message of the member of the IE team indicating that positive features were not to be overshadowed by continuous mention to students' difficulties. In the continuation she made an interesting reflection about the changes in students' knowledge provided by the 2007 mathematics syllabus:

Tânia: And it is the fact that they already represent equivalent fractions.

Marisa: They represent what?

Tânia: For example, in the past [before 2007], when they arrived here, we had to begin by all this phase, because they know what was ¼, ½, but not more. No, now they already know what is 3/8, 3/5, so…....

Marisa: In questions 1 and 2, they represent with equivalent fractions?

Tânia: Yes, yes.

Inês: So they come more developed.

Tânia: So, this first phase I think that we need to go ahead because we have to assume that this is learnt, because we see that this was worked in class. I have many… For example, here, they write the fraction but they write 1/2 in all; so they decided that instead of placing 4/8, 3/6, they put 1/2 in all. But, so, it is correct, it is one half, it is the equivalent fraction.

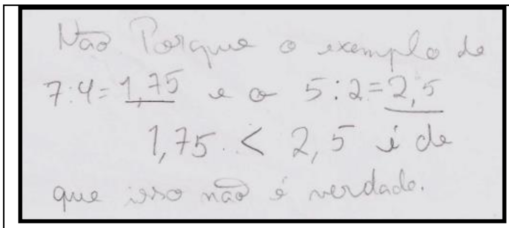
With this intervention, Tânia reflects about the changes that need to be made in teachers' practice in consequence of the changes introduced by the 2007 syllabus, as students got to learn about equivalent fractions through grades 1-4.

In the beginning of the discussion, teachers' comments systematically focused more on students' difficulties than identifying what they were able to do. However, with systematic encouragement from the IE team to focus on positive surprises, the teachers began to note and comment on interesting aspects of students' work. The fact that Marisa went to observe a class of one of the participating teachers and had some examples from this class provided an important support to the notion that there were indeed interesting things to mention. In this session we see the teachers reflecting with more confidence on issues related to the influence of curriculum documents on their teaching activity, recognizing that because of curriculum changes they can rely on the students' knowledge about equivalent fractions.

Recognizing different kinds of tasks and noticing students' reasoning

Session 4 included a discussion about the nature of tasks. We began by briefly presenting the main features of exercises, problems, explorations and investigations, referring to several examples. Tânia present a reflection about the distinction among problems and exercises based on examples from her own practice. She shows to understand that what students know is critical to establish this distinction, so that a task that is a problem for a student at a given point later may become a simple exercise. It is also noticeable that the teachers put a high value in students' work in explorations. Francisca and Luísa recall an experience from her own practice in which the students explored, with manipulative materials, the sum of the angles of a triangle. They referred that such discovery was very important for students that "would no longer forget".

In this session there was also some discussion about reasoning processes and analysis of students' solutions. We began by addressing the notions of generalization and justification which the teachers seemed to have not much trouble in appropriating. Observing the students solutions (figures 1 and 2), Tânia and Inês are able to note easily both generalizations and justifications.



No, because the example of
7:4=1,75 and 5:2=2,5
1,75 < 2,5 and this
means the it [the statemen]) is not true.

Figure 1. Identifying a justification

Tânia: It is more a justification; he searched an example.

Observing the response of the students in figure 1, Tânia quickly identifies that the students use a counter-example to refute a statement and, therefore, they were providing a justification. Analysing the solution presented in figure 2, Inês identifies the justification in a): "Here, this is a justification". She recognised that the students used another representation to verify the statement.

Quickly Tânia moved to b) and identifies the generalization:

Tânia: But the, in the other, they have already another little generalization.

Joana: In the other they have another small generalization. That is not so small.

Tânia: It is not for all [students].

Is it $\frac{2}{4} = \frac{8}{16}$? *Yes.* *0.5=0.5* Provide one or more justifications to your previous answer. $\frac{2}{4} = 0{,}5$; $\frac{8}{16} = 0{,}5$ $0{,}5 = 0{,}5$	*A number divided by its double is equal to 0.*5

Figure 2 – Identifying a generalization.

Given this recognition and the way Tânia states it, we decided to stress not just her discovery but also the work of the students so that the teachers understand the nature of this work and the importance of including it in their classes.

CONCLUSION

The results show that the teachers began their participation in the lesson study very suspicious about this activity. However, the fact that they could work on mathematical tasks and analyse students' solutions, with reference to their own experience, promoted their quick involvement. The carefully planned sessions and the environment of open questioning of issues, respect for everyone's ideas, argumentation supported on classroom data and also on research results yielded an atmosphere of interest and trust of a collaborative setting (Boavida & Ponte, 2002). As teachers began realizing that they had much to learn about their students' learning and their own learning on this topic, there were no more suggestions that we should be doing something else.

The whole experience of the lesson study is much richer than what we can glimpse in just a few sessions. However, we see the value of focusing on students' reasoning in working in mathematical tasks, with the possibility for teachers to notice important features of tasks that make them simple exercises or more engaging problems or explorations (Ponte, 2005; Skovsmose, 2001), as well as features of reasoning processes such as justification and generalization (Lannin, Ellis, & Elliot, 2011; Ponte, Mata-Pereira, & Henriques, 2012). Anticipating possible difficulties of students and looking at what they actually do in the classroom are key features of lesson study (Alston, Pedrick, Morris, & Basu, 2011) that proved to be very effective in leading the teachers to reflect and consider changes in their classroom practice. The attention to affective issues, regarding the way teachers are invited to become involved in lesson studies, the collaborative environment provided, and the activity that they have opportunity to undertake also appear as critical design factors (Loucks-Horsley, Hewson, Love, & Stiles, 1998) for the success of this activity.

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